Dual Tree Discrete Wavelet Transform (DT-DWT) and Wiener Filter- A Review

Ravindra Mishra
M.Tech scholar, Ojaswini Institute of Management and Technology, Damoh, Madhya Pradesh, India.

Anand Vardhan Bhalla
Assistant Professor, Electronics and Communication Engineering Department, Ojaswini Institute of Management and Technology, Damoh, Madhya Pradesh, India.

Abstract – Visual information transmitted in the form of digital images is becoming a major method of communication in the modern age, but the image obtained after transmission is often corrupted with noise. The received image needs processing before it can be used in applications. Image denoising involves the manipulation of the image data to produce a visually high quality image. Selection of the denoising algorithm is application dependent. Hence, it is necessary to have knowledge about the noise present in the image so as to select the appropriate denoising algorithm. The wavelet based approach finds applications in denoising images corrupted with Gaussian noise. In the case where the noise characteristics are complex, the multifractal approach can be used. There is a wide range of applications in which denoising is important. Examples are medical image/signal analysis, data mining, radio astronomy and there are many more. The multi-resolution analysis performed by the wavelet transform has proved to be particularly efficient in image denoising. Since the early use of the classical orthonormal wavelet transform for removing additive white Gaussian noise through thresholding, a lot of work has been done leading to some important observations. *Firstly,* Better performances can be achieved with shiftinvariant transformations. *Secondly,* Directionality of the transform is important in processing geometrical images. *Finally,* Further improvements can be obtained with more sophisticated thresholding functions which incorporate inter-scale and intra-scale dependencies. Various methods have been attempted to take advantage of these observations, the dual tree discrete wavelet transform (DT-DWT) that uses two sets of critically sampled filters that form Hilbert transform pairs is popular.

Index Terms – WT (Wavelet transform), DT-DWT (Dual Tree Discrete Wavelet Transform), Wiener filter, Image processing, Noise.

1. INTRODUCTION

It is well known fact that signals do not exist without noise, which may be negligible (i.e. high SNR) under certain conditions. However, there are many cases in which the noise corrupts the signals in a significant manner, and it must be removed from the data in order to proceed with further data analysis. The process of noise removal is generally referred to as signal denoising or simply denoising. There is a wide range of applications in which denoising is important. Examples are medical image/signal analysis, data mining, radio astronomy and many more. Each application has its special requirements. For example, noise removal in medical signals requires specific care, since denoising which involves smoothing of the noisy signal (e.g., using low-pass flter) may cause the loss of fine details.

The optimization criterion, according to which the performance of a denoising algorithm is measured, is usually taken to be Peak Signal to Noise Ratio (PSNR), Mean Squared Error (MSE) and Structural Similarity Index (SSIM) between the original signal and its reconstructed version. This common criterion is used mostly due its computational simplicity. Moreover, it usually leads to expressions which can be dealt with analytically. However, this criterion may be inappropriate for some tasks in which the criterion is perceptual quality driven, though perceptual quality assessment itself is a difficult problem, especially in the absence of the original signal.

2. HISTORICAL OVERVIEW OF IMAGE DENOISING RESEARCH

Image Denoising has remained a basic problem in the field of image processing. Wavelets give a superior performance in image denoising due to properties such as sparsity and multiresolution structure. With Wavelet Transform gaining popularity in the last two decades various algorithms for denoising in wavelet domain were introduced. The focus was shifted from the Spatial and Fourier domain to the Wavelet transform domain. Ever since Donoho's Wavelet based thresholding approach was published in 1995, there was a surge in the denoising papers being published. Although Donoho's concept was not revolutionary, his methods did not require tracking or correlation of the wavelet maxima and minima across the different scales as proposed by Mallat. Thus, there was a renewed interest in wavelet based denoising techniques since Donoho demonstrated a simple approach to a difficult problem. Researchers published different ways to compute the parameters for the thresholding of wavelet coefficients. Data adaptive thresholds were introduced to achieve optimum value of threshold. Later efforts found that substantial improvements in perceptual quality could be obtained by translation invariant methods based on thresholding of an Undecimated Wavelet Transform. These thresholding techniques were applied to the nonorthogonal wavelet coefficients to reduce artifacts. Multiwavelets were also used to achieve similar results. Probabilistic models using the statistical properties of the wavelet coefficient seemed to outperform the thresholding techniques. Recently, much effort has been devoted to Bayesian denoising in Wavelet domain. Hidden Markov Models and Gaussian Scale Mixtures have also become popular and more research continues to be published. Tree Structures ordering the wavelet coefficients based on their magnitude, scale and spatial location have been researched. Data adaptive transforms such as Independent Component Analysis (ICA) have been explored for sparse shrinkage. The trend continues to focus on using different statistical models to model the statistical properties of the wavelet coefficients and its neighbors. Future trend will be towards finding more accurate probabilistic models for the distribution of non-orthogonal wavelet coefficients.

The multi-resolution analysis performed by the wavelet transform has proved to be particularly efficient in image denoising. Since the early use of the classical orthonormal wavelet transform for removing additive white Gaussian noise through thresholding, a lot of work has been done leading to some important observations:

- Better performances can be achieved with shiftinvariant transformations.
- Directionality of the transform is important in processing geometrical images.
- Further improvements can be obtained with more sophisticated thresholding functions which incorporate inter-scale and intra-scale dependencies.

Various methods have been attempted to take advantage of these observations such as the undecimated discrete wavelet transform (UDWT) (becomes shift-invariant through the removal of the down sampling found in the DWT), the double density discrete wavelet transform (DDDWT) (which uses oversampled filters), the dual tree discrete wavelet transform (DT-DWT) (uses two sets of critically sampled filters that form Hilbert transform pairs) and the double density dual tree wavelet transform (DDDTDWT) (uses two sets of oversampled filters forming complex sub bands to give spatial feature information along multiple directions). For estimating the optimal threshold many procedures have been developed, such as VisuShrink, BayesShrink, SUREshrink, NeighSURE etc. These methods model the denoising problem assuming various distributions for the signal coefficients and noise components. The optimal threshold is determined based on estimators such as maximum apriority (MAP), maximum absolute deviation (MAD), maximum likelihood estimation (MLE) etc.

3. DUAL TREE DISCRETE WAVELET TRANSFORM (DT-DWT)

The classical discrete wavelet transform (DWT) provides a means of implementing a multiscale analysis, based on a critically sampled filter bank with perfect reconstruction. However, questions arise regarding the good qualities or properties of the wavelets and the results obtained using these tools, the standard DWT suffers from the following problems described as below:

1. *Shift sensitivity:* It has been observed that DWT is seriously disadvantaged by the shift sensitivity that arises from down samples in the DWT implementation.

2. *Poor directionality:* An m-dimension transform (m>1) suffers poor directionality when the transform coefficients reveal only a few feature in the spatial domain.

3. *Absence of phase information:* Filtering the image with DWT increases its size and adds phase distortions; human visual system is sensitive to phase distortion. Such DWT implementations cannot provide the local phase information.

In other applications, and for certain types of images, it is necessary to think of other, more complex wavelets, which gives a good way, because the complex wavelets filters which can be made to suppress negative frequency components. The complex wavelet transform has improved shift-invariance and directional selectivity. This implementation uses consists in analyzing the signal by two different DWT trees, with filters chosen so that at the end, the signal returns with the approximate decomposition by an analytical wavelet. The dual-tree structure has an extension of conjugate filtering in 2-D case. Because of the existence of two trees the second noise coefficients moments from such decomposition can be precisely characterized. The DT-DWT ensures filtering of the results without distortion and with a good ability for the localization function and the perfect reconstruction of signal. In the noise study, as with any redundant frame analysis, when a stationary noise, even if white, is subject to a dual decomposition tree, statistical dependencies appear between coefficients, because of the existence of two trees, it appears that the second noise coefficients moments from such decomposition can be precisely characterized. We observe a de-correlation between primal and dual coefficients located at the same spatial position and an inter-scale correlation, which allows us to choose between several estimators, taking this phenomenon into account. If we consider an image degraded by centered, additive Gaussian noise with a spectral density, the decomposition coefficients are also affected by that same noise as part of the linearity property. With this advantage we can choose an appropriate estimator for de-noising. In the case of DT-DWT the mathematical expression for a signal observed at point whose coordinates (x,y) in the image is modeled as follows:

$$g(x,y) = f(x,y) + \varepsilon(x,y)$$

With g(x,y) , f(x,y) and ε(x,y) are respectively the noise coefficient, the original coefficient, and the Gaussian independent noise. After applying the DT-DWT on g(x,y) we obtain:

$$g_\eta(x,y) = f_\eta(x,y) + \varepsilon_\eta(x,y)$$

Where, $g_\eta(x,y)$, ,$f_\eta(x,y)$ and $\varepsilon_\eta.(x,y)$ denote $(x,y)^{th}$ wavelet coefficient at level of a particular detail subband of the DT-DWT of g, f, and ε , respectively and η (η= 1,2…,J).

The dual-tree complex DWT of a signal x is implemented using two critically-sampled DWTs in parallel on the same data. The transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed is a specific way, then the sub band signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and sub band signals of the lower DWT can be interpreted as the imaginary part.

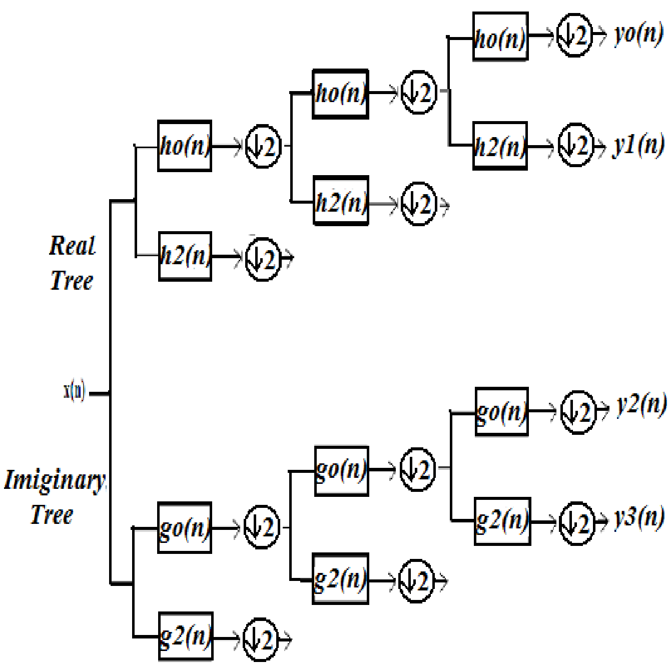

Figure 3.1: Implementation of Dual-Tree Discrete Wavelet Transform.

Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be an approximate Hilbert transform of the wavelet associated with the lower DWT. When designed in this way, the dual-tree complex DWT is nearly shift-invariant, in contrast with the critically-sampled DWT. Moreover, the dual-tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. The dual-tree DWT outperforms the critically sampled DWT for applications like image denoising and enhancement.

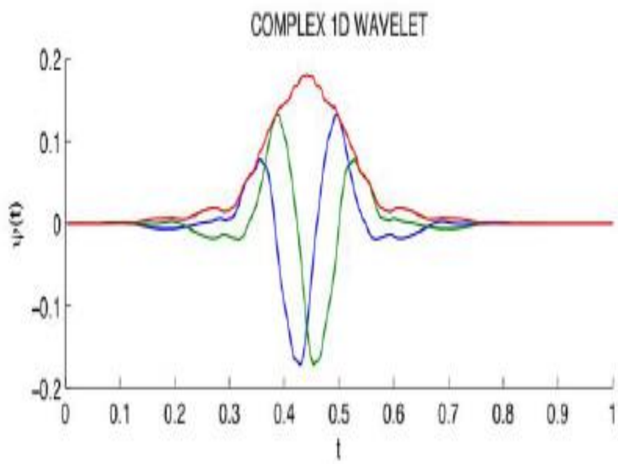

Figure 3.2: Complex 1 D Wavelet.

4. WIENER FILTER

This filter was proposed by Norbert Wiener during the 1940s. The discrete-time equivalent of Wiener's work was derived independently by Andrey Kolmogorov and published in 1941. Hence the theory is often called the Wiener–Kolmogorov filtering theory (cf. Kriging). The Wiener filter was the first statistically designed filter to be proposed and subsequently gave rise to many others including the Kalman filter. The Wiener filter is the MSE-optimal stationary linear filter for images degraded by additive noise and blurring. Wiener filters are usually applied in the frequency domain. Given a degraded image x(n,m), one takes the Discrete Fourier Transform (DFT) to obtain X(u,v). The original image spectrum is estimated by taking the product of X(u,v) with the Wiener filter G(u,v):

$$\hat{S}(u,v) = G(u,v)X(u,v)$$

The inverse DFT is then used to obtain the image estimate from its spectrum. The Wiener filter is defined in terms of these spectra:

H(u,v) = Fourier transform of point spread function(PSF).

$P_S(u,v)$ = Power spectrum of the signal process obtained by taking the fourier transform of the signal autocorrelation.

$P_n(u,v)$ = Power spectrum of the noise process obtained by taking the fourier transform of the noise autocorrelation.

The Wiener filter is:

$$G(u,v) = \frac{H^*(u,v)P_s(u,v)}{|H(u,v)|^2 P_s(u,v) + P_n(u,v)}$$

Dividing through by P_S makes its behavior easier to explain:

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{P_n(u,v)}{P_s(u,v)}}$$

The term P_n/P_S can be interpreted as the reciprocal of the signal-to-noise ratio. Where the signal is very strong relative to the noise, P_n/P_S is approximately equal to zero and the Wiener filter becomes $H^{-1}(u,v)$ - the inverse filter for the PSF. Where the signal is very weak, P_n/P_S and $G(U,V)$ tends towards zero.

For the case of additive white noise and no blurring, the Wiener filter simplifies to:

$$G(u,v) = \frac{P_s(u,v)}{P_s(u,v) + \sigma_n^2}$$

where σ_n^2 is the noise variance.

Wiener filters are unable to reconstruct frequency components which have been degraded by noise. They can only suppress them. Also, Wiener filters are unable to restore components for which $H(u,v)=0$. This means they are unable to undo blurring caused by bandlimiting of $H(u,v)$. Such bandlimiting occurs in any real-world imaging system.

Obtaining P_S can be problematic. One can assume that P_S has a parametric shape, for example exponential or Gaussian. Alternately, P_S can be estimated using images representative of the class of images being filtered.

Wiener filters are comparatively slow to apply, since they require working in the frequency domain. To speed up filtering, one can take the inverse FFT of the Wiener filter $G(u,v)$ to obtain an impulse response $g(n,m)$. This impulse response can be truncated spatially to produce a convolution mask. The spatially truncated Wiener filter is inferior to the frequency domain version, but may be much faster. The Wiener filter has a variety of applications in signal processing, image processing, control systems, and digital communications. These applications generally fall into one of four main categories:

- System identification.
- Deconvolution.
- Noise reduction.
- Signal detection.

For example, the Wiener filter can be used in image processing to remove noise from a picture.

The choice of Wiener filter order affects:

(a) The ability of the filter to remove distortions and reduce the noise.

(b) The computational complexity of the filter.

(c) The numerical stability of the of the Wiener solution.

The choice of the filter length also depends on the application and the method of implementation of the Wiener filter. For example, in a filter-bank implementation of the Wiener filter for additive noise reduction, the number of filter coefficients is equal to the number of filter banks. A filter-bank implementation of a Wiener filter number of filter banks is between 16 to 64. On the other hand for many applications, a direct implementation of the time-domain Wiener filter requires a larger filter length say between 64 and 256 taps. A reduction in the required length of a time-domain Wiener filter can be achieved by dividing the time domain signal into N sub-band signals. Each sub-band signal can then be decimated by a factor of N. The decimation results in a reduction, by a factor of N, in the required length of each sub-band Wiener filter.

5. CONCLUSION

The DT-DWT outperforms in comparison to DWT for applications like image denoising and enhancement. Wiener filter is used to filter the noise present in the image. From the above study it can be concluded that both DTDWT and Wiener filter shows excellent performance when the image is affected by the additive white Gaussian noise.

REFERENCES

[1]. Lahcène Mitiche and Amel Baha Houda ADAMOU-Mitiche and Hilal Naimi, *"Medical image denoising using Dual Tree Complex Thresholding Wavelet Transform"*, 2013 IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT).

[2]. Varun P. Gopi, Pavithran M., Nishanth T., Balaji S., Rajavelu v., P. Palanisamy, *"Undecimated double density dual tree wavelet transform based image denoising using a subband adaptive threshold"*, 978-1-4799-2900-9/14/$31.00 ©2014 IEEE.

[3]. Md. Motahar Hossain Mishu, A. B. M. Aowlad Hossain, Md. Ehsan Ahmed Emon ,*"Denoising of ECG Signals Using Dual Tree Complex Wavelet Transform"*, 2014 17th International Conference on Computer and Information Technology*(ICCIT)*.

[4]. Chandrika Saxena , Prof. Deepak Kourav, *"Noises and Image Denoising Techniques: A Brief Survey"*, International Journal of Emerging Technology and Advanced Engineering, ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 4, Issue 3, March 2014.

[5]. Anutam and Rajni, *"Performance Analysis Of Image Denoising With Wavelet Thresholding Methods For Different Levels Of Decomposition"*, The International Journal of Multimedia & Its Applications (IJMA) Vol.6, No.3, June 2014.

[6]. Chandrika Saxena , Prof. Deepak Kourav, *"Noises and Image Denoising Techniques: A Brief Survey"* International Journal of Emerging Technology and Advanced Engineering, ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 4, Issue 3, March 2014.

[7]. Mr. Rahul K. Sarawale and Dr. S. R. Chougule, *"Noise Removal Using Double-Densit Dual-Tree Complex DWT*", Proceedings of the 2013 IEEE Second International Conference on Image Information Processing (ICIIP-2013).

[8]. Iram Sami, Abhishek Thakur, Rajesh Kumar, *"Image Denoising for Gaussian Noise Reduction in Bionics Using DWT Technique",* IJECT Vol. 4, Issue April - June 2013.

[9]. Varun P. Gopi, Pavithran M., Nishanth T., Balaji S., Rajavelu V., P. Palanisamy *"Image denoising based on Undecimated Double Density Dual tree Wavelet Transform and Modified Firm Shrinkage",* 978-0-7695-5127-2/13 © 2013 IEEE DOI 10.1109/ADCONS.2013.38.

[10]. Namrata Dewanga, Agam Das Goswam*, "Image Denoising Using Wavelet Thresholding Methods",* International Journal of Engineering Sciences & Management. Int. J. of Engg. Sci. & Mgmt. (IJESM), Vol. 2, Issue 2: April-June: 2012, 271 -275.

[11]. Kanwaljot Singh Sidhu , Baljeet Singh Khaira , Ishpreet Singh Virk, *" Medical Image Denoising In The Wavelet Domain Using Haar And DB3 Filtering",* International Refereed Journal of Engineering and Science (IRJES) ISSN (Online) 2319-183X, (Print) 2319-1821 Volume 1, Issue 1 (September 2012), PP.001-008.

[12]. Liu Fang Biao Yang " *UAV Image Denoising Using Adaptive Dual-Tree Discrete Wavelet Packets Based on Estimate the Distributing of the Noise"* 978-1-4673-1398-8/12/ ©2012 IEEE

[13]. Akhilesh Bijalwan, Aditya Goyal, Nidhi Sethi, *"Wavelet Transform Based Image Denoise Using Threshold Approaches",* International Journal of Engineering and Advanced Technology (IJEAT) ISSN: 2249 – 8958, Volume-1, Issue-5, June 2012.

[14]. V. Naga Prudhvi Raj, T Venkateswarlu, "*Denoising of medical images using dual tree complex wavelet transform"*, Procedia Technology 4 (2012) 238-244, C3IT-2012.

[15]. B. Ismail, Anjum Khan, " *Image De-noising with a New Threshold Value Using Wavelets ",* Journal of Data Science 10 (2012), 259-270.

[16]. Gurmeet Kaur, Rupinder Kaur, "*Image De-Noising using Wavelet Transform and various Filters*", International Journal of Research in Computer Science ISSN: 2249-8265 Volume 2 Issue 2 (2012) pp. 15-21.

[17]. S. Satheesh, K. Prasad, *"Medical image denoising using adaptive Threshold based on contourlet transform"*, ACIJ, Vol.2, No.2, March 2011.

[18]. S.Kalavathy and R.M.Suresh, *"Analysis of Image Denoising using Wavelet Coefficient and Adaptive Subband Thresholding Technique",* IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 6, No 1, November 2011,ISSN: 1694-0814.

[19]. Miss. Pallavi Charde, *" A Review On Image Denoising Using Wavelet Transform And Median Filter Over AWGN Channel",* International Journal Of Technology Enhancements And Emerging Engineering Research, Vol 1, Issue 4, ISSN 2347-4289.

[20]. Kekre, H. B. Sectorization of Full Kekre "Wavelet *Transform for Feature extraction of Color Images*", International Journal of Advanced Computer Science and Applications - IJACSA, 2(2), 69-74. (2011).

[21]. Suresh Kumar, Papendra Kumar, Manoj Gupta, Ashok Kumar Nagawat, *"Performance Comparison of Median and Wiener Filter in Image De-noising",* International Journal of Computer Applications (0975 – 8887) Volume 12– No.4, November 2010.

[22]. Sasikala, P. "*Robust R Peak and QRS detection in Electrocardiogram using Wavelet Transform*", International Journal of Advanced Computer Science and Applications - IJACSA, 1(6), 48-53(2010).

[23]. Yan-Li Liu, Jin Wang, Xi Chen, Yan-Wen Guo, *"A Robust and Fast Non-Local Means Algorithm for Image Denoising",* Journal Of Computer Science And Technology 23(2): 270–279 Mar. 2008.

[24]. Rajat Singh, D.S. Meena, *"Image Denoising Using Curvelet Transform*" Department of Computer Science and Engineering National Institute of Technology, Rourkela.

[25]. Adrian E. Villanueva- Luna1, Alberto Jaramillo-Nuñez1, Daniel Sanchez-Lucero1, Carlos M. Ortiz-Lima1, J. Gabriel Aguilar-Soto1, Aaron Flores-Gil2 and Manuel May-Alarcon2, *" De-Noising Audio Signals Using MATLAB Wavelets",* Instituto Nacional de Astrofisica, Optica y Electronica (INAOE) ,Universidad Autonoma del Carmen (UNACAR)Mexico.

[26]. Stephen Wolf, Margaret Pinson, *"Algorithm for Computing Peak Signal to Noise Ratio (PSNR) of a Video Sequence with a Constant Delay",* Geneva, February 2-6, 2009.

[27]. D.Gnanadurai and Y.Sadasivam, *"An Efficient Adaptive Thresholding Techniques for Wavelet Based Image Denoising*", World Academy of Science, Engineering and Technology 20, pp. 1121-1126,2008

[28]. S.Arivazhagan, S.Deivalakshmi, K.Kannan, *"Performance Analysis of Image Denoising System for different levels of Wavelet decomposition*", International Journal Of Imaging Science And Engineering (IJISE) ,VOL.1, NO.3, pp. 104-107, July 2007.

[29]. H. Rabbani, M. Vafadust, and S. Gazor "*Image denoising based on a mixture of laplacian distributions with local parameters in complex wavelet domain"*, in Proc. Int. Conf. Image Process., Atlanta, Oct. 2006, pp. 2597–2600.

[30]. I.W. Selesnick, R.G. Baraniuk, and N.C. Kingsbury, *"The dual-tree complex wavelet transform",* Signal Processing Magazine, IEEE, 2005., 22 (6): p. 123-151.

[31]. J. E. Fowler, "*The redundant discrete wavelet transform and additive noise"* IEEE Signal Processing Letters pp. 629-632, 2005.

[32]. L. Sendur and I. W. Selesnick , "*Bivariate shrinkage functions for wavelet-based denoising exploiting interscale dependency*", IEEE Trans. Signal Process., vol. 50, no. 11, pp. 2744–2756, Nov. 2002.

[33]. Y. Yang, *"Image denoising using wavelet thresholding techniques", 2005.*

[34]. Z. Wang, A. C. Bovik*, "Image Quality and structural similarity",* IEEE Transactios on Image Processing, vol. 13,no. 1,2004.

[35]. A. F. Abdelnour and I. W. Selesnick, *"Symmetric nearly orthogonal and orthgonal nearly symmetric wavelets,*" The Arabian J. for Sci. and Eng., vol. 29, pp. 3-16, Dec. 2004.

[36]. D. Cho, *"Image denoising using wavelet transforms,"* Master Degree Thesis, Dept. Comput. Sci., Concordia Univ., Canada, 2004.

[37]. I. W. Selesnick, "*The double-density dual-tree DWT*," IEEE Trans. Signal Process., vol. 52(5), pp. 1304-1314, May 2004.

[38]. G.Y. Chen∗, T.D. Bui, A. Krzy *zak "Image denoising with neighbor dependency and customized wavelet and threshold",* Department of Computer Science, Concordia University, 1455 De Maisonneuve West, Montreal, Que., Canada H3G1M8 Received 30 May 2003, received in revised form 19 April 2004; accepted 11 May 2004.